AN ANALYTICAL APPROACH TO LOCAL SOUND FIELD SYNTHESIS USING LINEAR ARRAYS OF LOUDSPEAKERS

Jens Ahrens and Sascha Spors

Quality and Usability Lab, Deutsche Telekom Laboratories, Technische Universität Berlin
Ernst-Reuter-Platz 7, 10587 Berlin, Germany
{jens.ahrens,sascha.spors}@telekom.de

ABSTRACT

Methods like Wave Field Synthesis aim at the synthesis of a given desired sound field over a large receiver area. Practical limitations lead to considerable artifacts commonly referred to as spatial aliasing. Above a given frequency these artifacts are apparent anywhere in the receiver area when linear arrays of secondary sources are considered. This paper presents an analytical approach based on the Spectral Division Method which achieves an accuracy of the synthesized sound field which is significantly higher than in conventional approaches in a limited target zone. This local increase in accuracy is achieved via a manipulation of the spatial bandwidth of the secondary source driving function.

Index Terms— Local sound field synthesis, Spectral Division Method, spatial aliasing, spatial Fourier transform

1. INTRODUCTION

Sound field synthesis methods for audio presentation like Wave Field Synthesis (WFS) [1] and Near-field Compensated Higher Order Ambisonics [2] have received considerable attention during the last years. This may be attributed to the fact that, unlike e.g. stereophony, these methods theoretically provide the potential to evoke a plausible aural perspective over an extended receiver area. Practical limitations, however, lead to inaccuracies and artifacts which are commonly summarized by the term *spatial aliasing* [3]. The perceptual consequences of these artifacts are hardly known. For static scenarios it is assumed that the artifacts lead to varying degrees of coloration [4]; for dynamic scenarios they can lead to more severe degradation [5].

The spatial distribution of the artifacts is largely dependent on the geometry of the secondary source contour employed and on the spatial bandwidth of the secondary source driving signals [6]. When linear secondary source distributions are considered, the artifacts are distributed over the entire potential receiver area.

In this paper, we present an analytical approach to sound field synthesis employing linear secondary source distributions which leads to a significant increase of accuracy in a predefined target area of limited size. For convenience, we illustrate the approach using the Spectral Division Method (SDM). The latter is available for planar and linear secondary source distributions [7]. Exemplarily, only linear distributions are treated in this paper. The extension of the results to planar distributions is straightforward.

A comparable approach using circular distributions of secondary sources can be found in [8].

Numerical solutions like [9, 10] can also be employed for the purpose of sound field synthesis with linear secondary source distributions and, if the optimization criterion is set appropriately, are capable of creating a limited zone of increased accuracy. However, such numerical approaches are computationally costly and are intransparent with respect to physical limitations and properties of the synthesized sound fields.

2. THE SPECTRAL DIVISION METHOD

For convenience, a continuous linear secondary source distribution is assumed which is located along the x-axis in the following. Refer to Fig. 1.
For this setup, the synthesis equation is given by [7]

$$S(\mathbf{x},\omega) = \int_{-\infty}^{\infty} D(\mathbf{x}_0,\omega) \cdot G(\mathbf{x}-\mathbf{x}_0,\omega)\, dx_0 \,. \qquad (1)$$

$D(\mathbf{x}_0,\omega)$ denotes the driving function of the secondary source located at $\mathbf{x}_0 = [x_0 \ 0 \ 0]^T$ and $G(\mathbf{x}-\mathbf{x}_0,\omega)$ its spatio-temporal transfer function. In order that (1) holds $G(\mathbf{x}-\mathbf{x}_0,\omega)$ has to be invariant with respect to translation along the linear secondary source contour.

Equation (1) can be interpreted as a convolution along the x-axis and the convolution theorem

$$\tilde{S}(k_x,y,z,\omega) = \tilde{D}(k_x,\omega) \cdot \tilde{G}(k_x,y,z,\omega) \qquad (2)$$

holds [11]. The secondary source driving function $\tilde{D}(k_x,\omega)$ in wavenumber domain is thus given by

$$\tilde{D}(k_x,\omega) = \frac{\tilde{S}(k_x,y,z,\omega)}{\tilde{G}(k_x,y,z,\omega)} \,. \qquad (3)$$

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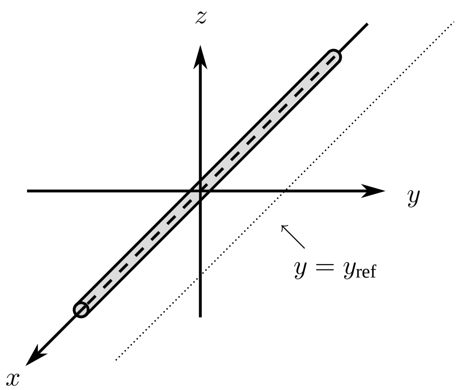


Fig. 1: Illustration of the setup of a linear secondary source situated along the x-axis. The secondary source distribution is indicated by the grey shading and has infinite extent. The target half-plane is the half-plane bounded by the secondary source distribution and containing the positive y-axis. The thin dotted line indicates the reference line (see text).

In the above derivation, we intentionally assumed $D(x,\omega)$ to be exclusively dependent on x because x is the only degree of freedom in the position of the secondary sources. However, generally $D(x,\omega)$ will be dependent on the position of the receiver. This is mathematically reflected by the fact that y and z do not cancel out in (3) [7].

It is not surprising that we are not able to synthesize arbitrary sound fields since the secondary source setup is capable of creating wave fronts that propagate away from it. We will treat this circumstance in an intuitive way in the following. Refer to [7] for a rigorous derivation.

The propagation direction of the synthesized sound field can generally only be correct inside one half-plane bounded by the secondary source distribution. We term this half-plane *target half-plane*. The synthesized sound field anywhere else in space is a byproduct the properties of which are determined by the secondary source driving function $D(x,\omega)$ and the radiation characteristics of the secondary sources in the respective direction. For convenience, we aim at synthesizing a given desired sound field inside that half of the horizontal plane which contains the positive y-axis. We therefore set $z = 0$.

Further treatment shows that the synthesized sound field will generally only be correct on a line parallel to the x-axis at distance $y = y_{\text{ref}}$ [7]. At locations off this reference line, the synthesized sound field generally deviates from the desired sound field in terms of amplitude, propagation direction, and near-field components.

Such a situation is termed *2.5-dimensional synthesis* [1] since the synthesis is neither purely two-dimensional nor purely three-dimensional but rather something in between. The properties of 2.5-dimensional synthesis are similar for all one-dimensional secondary source geometries [6].

In order to simplify the mathematical treatment, we restrict the validity of equations (1)–(3) to our reference line in the target half-plane, i.e. $z = 0$ and $y = y_{\text{ref}}$ (see Fig. 1).

Equation (3) is then given by

$$\tilde{D}(k_x,\omega) = \frac{\tilde{S}(k_x, y_{\text{ref}}, 0, \omega)}{\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)} \ . \tag{4}$$

Performing an inverse Fourier transform with respect to k_x on (4) yields the driving function $D(x,\omega)$ in temporal spectrum domain as

$$D(x,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\tilde{S}(k_x, y_{\text{ref}}, 0, \omega)}{\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)} \, e^{-ik_x x} \, dk_x \ . \tag{5}$$

In order that $D(x,\omega)$ is defined, $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$ may not exhibit zeros. For ill-posed $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$, regularization can be applied in practice in order to ensure a good behavior of its inverse. Refer to [12] for considerations on the incorporation of secondary source directivity.

In the remainder of this paper, the synthesis of the sound field of a virtual monopole sound source is considered and secondary monopoles are assumed. $\tilde{G}(k_x, y_{\text{ref}}, 0, \omega)$ then equals the free-field Green's function $\tilde{G}_0(k_x, y_{\text{ref}}, 0, \omega)$ and is given by [7]

$$\tilde{G}(k_x, y_{\text{ref}}, 0, \omega) = \tilde{G}_0(k_x, y_{\text{ref}}, 0, \omega) = \begin{cases} -\frac{i}{4} H_0^{(2)}\left(\sqrt{\left(\frac{\omega}{c}\right)^2 - {k_x}^2}\, y_{\text{ref}}\right) & \text{for } 0 \le |k_x| < \left|\frac{\omega}{c}\right| \\ \frac{1}{2\pi} K_0\left(\sqrt{{k_x}^2 - \left(\frac{\omega}{c}\right)^2}\, y_{\text{ref}}\right) & \text{for } 0 < \left|\frac{\omega}{c}\right| < |k_x| \end{cases} . \tag{6}$$

The spatial spectrum $\tilde{S}\left(k_x, y, z, \omega\right)$ of the sound field of a monopole sound source located at $\mathbf{x}_{\text{s}} = [x_{\text{s}} \ y_{\text{s}} \ 0]^T$ can be deduced from $\tilde{G}_0\left(k_x, y, z, \omega\right)$ given by (6) via the shift theorem of the Fourier transform as [11]

$$\tilde{S}\left(k_x, y, z, \omega\right) = e^{ik_x x_{\text{s}}} \tilde{G}_0\left(k_x, y - y_{\text{s}}, z, \omega\right) \ , \tag{7}$$

so that the driving function $\tilde{D}(k_x,\omega)$ (eq. (4)) explicitly reads

$$\tilde{D}(k_x,\omega) = e^{ik_x x_{\text{s}}} \times \begin{cases} \dfrac{H_0^{(2)}\left(\sqrt{\left(\frac{\omega}{c}\right)^2 - {k_x}^2}\,(y_{\text{ref}} - y_{\text{s}})\right)}{H_0^{(2)}\left(\sqrt{\left(\frac{\omega}{c}\right)^2 - {k_x}^2}\, y_{\text{ref}}\right)} & \text{for } 0 \le |k_x| < \left|\frac{\omega}{c}\right| \\ \dfrac{K_0\left(\sqrt{{k_x}^2 - \left(\frac{\omega}{c}\right)^2}\,(y_{\text{ref}} - y_{\text{s}})\right)}{K_0\left(\sqrt{{k_x}^2 - \left(\frac{\omega}{c}\right)^2}\, y_{\text{ref}}\right)} & \text{for } 0 < \left|\frac{\omega}{c}\right| < |k_x| \end{cases} . \tag{8}$$

In the following, a virtual point source at $\mathbf{x}_{\text{s}} = [0 \ -1 \ 0]^T$ m and $y_{\text{ref}} = 1$ m is considered. Eq. (8) for these parameters is depicted in Fig. 2(a) and the corresponding synthesized sound field in Fig. 3(a). Note that the latter was derived via a numerical Fourier transform since an analytical expression is not available.

3. SPATIAL DISCRETIZATION

The continuous secondary source distributions treated in Sec. 2 can not be implemented with today's available technology. It is rather such that continuous distributions have to be approximated by a finite number of discrete loudspeakers. This spatial discretization is typically modeled by a discretization of the corresponding driving function. Thus, a continuous distribution of secondary sources is assumed which is driven at discrete points. The essential benefit of this approach is the fact that all integral and convolution theorems exploited in Sec. 2 are still valid.

As shown in [3], equidistant sampling of the driving function of a continuous linear secondary source leads to repetitions of the spatial spectrum of the continuous driving function as

$$\tilde{D}_S(k_x, \omega) = \sum_{\eta=-\infty}^{\infty} \tilde{D}\left(k_x - \frac{2\pi}{\Delta x}\eta, \omega\right) \ . \tag{9}$$

As evident from (8) and Fig. 2(b), the continuous driving function $\tilde{D}\,(k_x, \omega)$ for a virtual spherical wave is not bandlimited with respect to k_x for unbounded $\omega = 2\pi f$. The discretization of the driving function $\tilde{D}\,(k_x, \omega)$ leads thus to an overlap and interference of the spectral repetitions above approximately 800 Hz for a secondary source spacing of $\Delta x = 0.2$ m (refer to Fig. 2(b)) and thus to a corruption of the synthesized sound field $S(\mathbf{x}, \omega)$. The latter is depicted in Fig. 3(b).

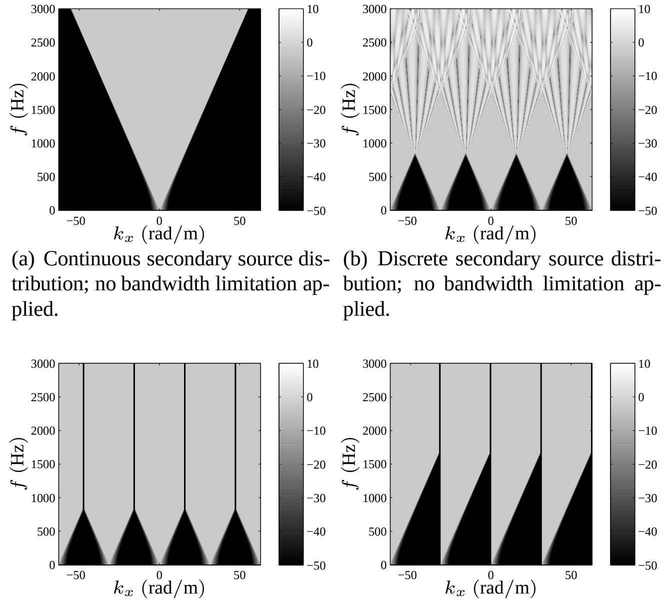
4. LOCAL SOUND FIELD SYNTHESIS

As mentioned in [3], overlap and interference of the spectral repetitions due to spatial discretization can be avoided via an appropriate spatial bandlimitation of the continuous driving function $D(x, \omega)$. The repetitions themselves can not be avoided. Note that bandlimiting the driving function is equal to choosing a spatially bandlimited desired sound field.

A bandlimitation of $\tilde{D}(k_x, \omega)$ with respect to k_x can be straightforwardly performed in the SDM by applying an appropriate window in k_x-domain. For simplicity, we choose a rectangular window.

Prevention of overlap of the spectral repetitions is achieved with a passband with a width of smaller than $\frac{2\pi}{\Delta x}$. For a secondary source spacing of $\Delta x = 0.2$ m, as employed in Fig. 3, this means that the passband has to be narrower or equal to approximately $31 \frac{\text{rad}}{\text{m}}$. Note that such a situation is termed *spatially narrowband synthesis* [6].

Limiting the spatial bandwidth of $\tilde{D}(k_x, \omega)$ in a manner symmetrical to $k_x = 0$, as depicted in Fig. 2(c), results in a synthesized sound field which is less corrupted by spatial discretization artifacts but the energy of which propagates primarily in direction perpendicular to the secondary source distribution. As a consequence, the amplitude of the synthesized sound field is significantly too low a certain locations in the target half-plane. However, the accuracy is increased around a straight line along $x = 0$.



(a) Continuous secondary source distribution; no bandwidth limitation applied.

(b) Discrete secondary source distribution; no bandwidth limitation applied.

(c) Discrete secondary source distribution; symmetrical bandwidth limitation applied.

(d) Discrete secondary source distribution; non-symmetrical bandwidth limitation applied.

Fig. 2: $20\log_{10}\left|\tilde{D}(k_x, \omega)\right|$ for continuous (Fig. 2(a)) and discrete linear secondary source distributions (Fig. 2(b)–(d)); $y_{\text{ref}} = 1$ m. With discrete distributions, the secondary source spacing is $\Delta x = 0.2$ m.

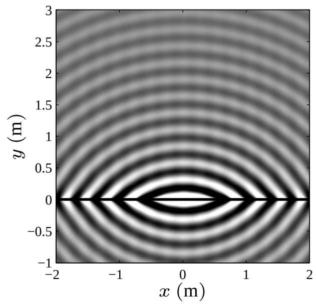
Limiting the spatial bandwidth of $\tilde{D}(k_x, \omega)$ in a manner which is not symmetrical to $k_x = 0$, as depicted in Fig. 2(d), allows for a steering of the straight line along which higher accuracy is achieved into a desired direction. The synthesis can therefore be optimized with respect to a given location of the receiver (e.g. the listener). By applying a time-varying center frequency of the passband, moving listeners can be tracked.

Due to the fact that the presented approach leads to a local increase of accuracy according methods are termed *local sound field synthesis* [6]. Of course, the local increase of accuracy comes by the cost of stronger artifacts outside the target zone. This circumstance is most evident in Fig. 3(d).

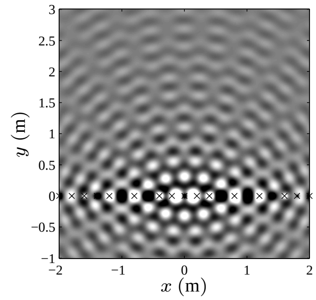
As mentioned above, the application of a spatial bandwidth limitation to reduce discretization artifacts has already been proposed in [3]. However, only passbands which are symmetrical to $k_x = 0$ are discussed and the properties of the synthesized sound fields are not investigated in detail.

5. CONCLUSIONS

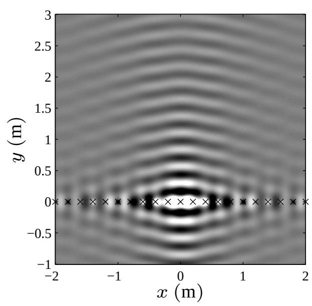
We presented an analytical approach to sound field synthesis employing linear arrays of secondary sources which achieves



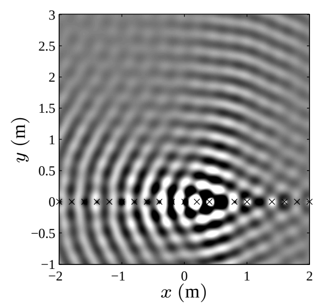
(a) Continuous secondary source distribution; no bandwidth limitation applied.



(b) Discrete secondary source distribution; no bandwidth limitation applied.



(c) Discrete secondary source distribution; symmetrical bandwidth limitation similar to Fig. 2(c) applied.



(d) Discrete secondary source distribution; non-symmetrical bandwidth limitation similar to Fig. 2(d).

Fig. 3: Cross-section through the horizontal plane for the synthesis of a virtual monopole source located at $\mathbf{x}_s = [0\ -1\ 0]^T$ m emitting a monochromatic signal of $f = 1300$ Hz; $\Re\{S(\mathbf{x}, \omega)\}$ is shown. In the continuous case, Fig. 3(a), the secondary source distribution is indicated by the black line; In the discrete cases in Fig. 3(b)–(d), the marks indicate the secondary sources. With discrete distributions, the secondary source spacing is $\Delta x = 0.2$ m.

higher accuracy than conventional approaches along a straight line the orientation of which can be steered. The local increase in accuracy was achieved via a limitation of the spatial bandwidth of the secondary source driving function in order to prevent overlap of the spectral repetitions which occur due to spatial discretization. Due to the local increase of accuracy such methods are termed *local sound field synthesis*.

The presented approach has been derived from the Spectral Division Method. The latter has been shown to be very convenient for local sound field synthesis due to its inherent space-frequency representation of the secondary source driving function which makes spatial bandlimitation straightforward. Of course, other methods like Wave Field Synthesis may also be employed.

In order to fully exploit the potential of the presented method optimal parameters of the passband have to be investigated, especially its width and shape of its slopes.

6. REFERENCES

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